

COMM 377 HMWK 1

Dr. H. S. Bhamra

1. An investor obtains utility  $u(x) = \frac{x^{1/3}}{1/3}$  from HC  $x$ . She faces the following gamble: a fair coin is tossed and if it shows heads she obtains HC 200 and if it shows tails, she gets HC 50. What is her certainty equivalent for this gamble? What is her risk-adjusted probability of receiving HC 50?
2. An investor obtains utility  $u(x) = \frac{x^{1/2}}{1/2}$  from HC  $x$ . She faces the following gamble: a fair coin is tossed and if it shows heads she obtains HC 200 and if it shows tails, she gets HC 50. What is her certainty equivalent for this gamble? What is her risk-adjusted probability of receiving HC 50?
3. Consider an investor who obtains utility  $u(x) = -e^{-ax}$  from HC  $x$ . This investor receives the random payoff (or gamble) HC  $\tilde{y}$ , where  $\tilde{y}$  is normally distributed with mean  $\mu$  and variance  $\sigma^2$ , i.e.  $\tilde{y} \sim N[\mu, \sigma^2]$ .

Find the investor's certainty equivalent for this gamble. Using your answer for the certainty equivalent, find  $b$  such that  $E[\tilde{z}] = CEQ[\tilde{y}]$ , where  $\tilde{z} \sim N[\mu - b, \sigma^2]$ .

You will need to use the following result. If  $\tilde{x} \sim N[m, s^2]$ , then  $E[e^{\tilde{x}}] = e^{m + \frac{1}{2}s^2}$ .

4. You purchase CAD 1M 12 months forward in exchange for USD. What is value of this forward contract?  
[The 1 year USD LIBOR rate is 4.8400% p.a., the 1 year CAD LIBOR rate is 3.9397% p.a, the spot rate is USD/CAD 0.8659 and the 1 year forward rate is USD/CAD 0.8734]
5. You purchase CAD 1M 6 months forward in exchange for USD. What is value of this forward contract? Check your answer by calculation.  
[The 6 month USD LIBOR rate is 4.7200% p.a., the 6 month CAD LIBOR rate is 3.7318% p.a, the spot rate is USD/CAD 0.8659 and the 6 month forward rate is USD/CAD 0.8701]

**Value of a forward contract at maturity**

6. 6 months ago, you bought CAD 1M 6 months forward in exchange for USD. What is the value of your forward contract today?  
[The spot rate is USD/CAD 0.8659 and 6 months ago the 6 month forward rate was USD/CAD 0.8634]
7. At date  $t$  you entered into a forward contract. The forward contract states that you will purchase FC 1 in exchange for HC at the forward rate  $\frac{HC}{FC} F_{t,T}$ . The exchange of cashflows will occur some time later at date  $T$ , where  $T > t$ . You jump into a time machine and just by chance you go forward in time to date  $T$  and your forward contract is expiring. What is the value of the forward contract at date  $T$ ?

### Value of an existing forward contract

8. One year ago you went long a 2 year forward contract on CAD 1M against the USD at the (old) forward rate of USD/CAD 0.8834. What is the value of your forward contract now? Compute your answer directly.

[The current 1 year USD LIBOR rate is 4.8400% p.a., the current 1 year CAD LIBOR rate is 3.9397% p.a, the spot rate now is USD/CAD 0.8659 and the current 1 year forward rate is USD/CAD 0.8734]

9. Same question—different methods. One year ago you went long a 2 year forward contract on CAD 1M against the USD at the (old) forward rate of USD /CAD 0.8834. What is the value of your forward contract now? Compute your answer twice—using the formulae below. Do both formulae give the same answer. If so, why?

[The current 1 year USD LIBOR rate is 4.8400% p.a., the current 1 year CAD LIBOR rate is 3.9397% p.a, the spot rate now is USD/CAD 0.8659 and the current 1 year forward rate is USD/CAD 0.8734]

$$V_{t+1} = \frac{S_{t+1}}{1 + r_{t+1,T}^*} - \frac{F_{t,T}}{1 + r_{t+1,T}}. \quad (1)$$

$$V_{t+1} = \frac{F_{t+1,T} - F_{t,T}}{1 + r_{t+1,T}}. \quad (2)$$

### Implied risk-neutral probabilities

10. You are given the following CAD/GBP forward rates: 1M 1.59578, 2M 1.60344, 3M 1.61262. The spot rate is 1.5882.

Construct a 3 period binomial tree. Making sensible assumptions for  $u$  and  $d = \frac{1}{u}$  compute the risk-neutral probabilities for up and down movements within the tree.